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Module: University Physics 2 (BDIC2008J)
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Problem Set 4

Conductors, Dielectrics and Capacitor

Problem 1. *An isolated conductor has net charge* $+10 \times 10^{-6}$C *a cavity with a particle of charge* $q = +3.0 \times 10^{-6}$C. *What is the charge on* ***(a)*** *the cavity wall and* ***(b)*** *the outer surface?*

Solution. **(a)** Consider a Gaussian surface that is completely within the conductor and surrounds the cavity. Since the electric field is zero everywhere on the surface, the net charge it encloses is zero. The net charge is the sum of the charge q in the cavity and the charge q_w on the cavity wall, so $q + q_w = 0$ and $q_w = -q = -3.0 \times 10^{-6}\text{C}$.

(b) The net charge Q of the conductor is the sum of the charge on the cavity wall and the charge q_s on the outer surface of the conductor, so $Q = q_w + q_s$ and

$$q_s = Q - q_w = (10 \times 10^{-6}\text{C}) - (-3.0 \times 10^{-6}\text{C}) = +1.3 \times 10^{-5}\text{C}$$

□

Problem 2. *The two metal objects in the figure below have net charges of* $+70\text{pC}$ *and* -70pC*, which result in a* $20V$ *potential difference between them.* ***(a)*** *What is the capacitance of the system?* ***(b)*** *If the charges are changed to* $+200\text{pC}$ *and* -200pC*, what does the capacitance become?* ***(c)*** *What does the potential difference become?*

Solution. **(a)** The capacitance of the system is

$$C = \frac{q}{\Delta V} = \frac{70\text{pC}}{20\text{V}} = 3.5\text{pF}$$

(b) The capacitance is independent of q; it is still 3.5pF.

(c) The potential difference becomes

$$\Delta V = \frac{q}{C} = \frac{200\text{pC}}{3.5\text{pF}} = 57\text{V}$$

□

Problem 3. *The plates of a spherical capacitor have radii* 38.0mm *and* 40.0mm. ***(a)*** *Calculate the capacitance.* ***(b)*** *What must be the plate area of a parallel-plate capacitor with the same plate separation and capacitance?*

Solution. **(a)** We use the Equation for spherical capacitor $C = 4\pi\varepsilon_0 \dfrac{ab}{b-a}$,

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} = \frac{(40.0\text{mm})(38.0\text{mm})}{(8.99\times10^{9}\text{N}\cdot\text{m}^2/\text{C}^2)(40.0\text{mm}-38.0\text{mm})} = 84.5\text{pF}$$

(b) Let the area required be S. Then $C = \varepsilon_0 S/(b-a)$, or

$$S = \frac{C(b-a)}{\varepsilon_0} = \frac{(84.5\text{pF})(40.0\text{mm}-38.0\text{mm})}{(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)} = 191\text{cm}^2$$

□

Problem 4. *What is the capacitance of a drop that results when two mercury spheres, each of radius* $R = 2.00\text{mm}$*, merge?*

Solution. Assuming conservation of volume, we find the radius of the combined spheres, then use $C = 4\pi\varepsilon_0 R$ to find the capacitance. When the drops combine, the volume is doubled. It is then $V = 2(4\pi/3)R^3$. The new radius R' is given by

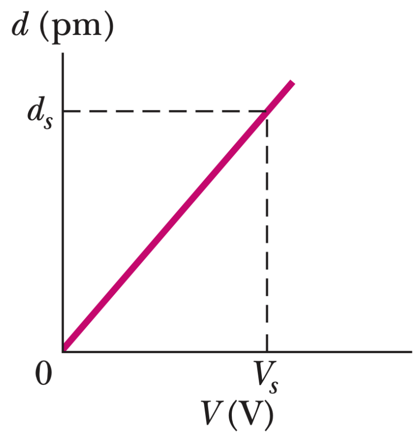
$$\frac{4\pi}{3}(R')^3 = 2\frac{4\pi}{3}R^3 \Rightarrow R' = 2^{1/3}R$$

The new capacitance is

$$C' = 4\pi\varepsilon_0 R' = 4\pi\varepsilon_0 2^{1/3}R = 5.04\pi\varepsilon_0 R$$

With $R = 2.00\text{mm}$, we obtain $C' = 5.04\pi(8.85 \times 10^{-12}\text{F/m})(2.00 \times 10^{-3}\text{m}) = 2.80 \times 10^{-13}\text{F}$. $\square$

Problem 5. *If an uncharged parallel-plate capacitor (capacitance C) is connected to a battery, one plate becomes negatively charged as electrons move to the plate face (area A). In the figure below, the depth d from which the electrons come in the plate in a particular capacitor is plotted against a range of values for the potential difference V of the battery. The density of conduction electrons in the copper plates is* 8.49×10^{28}electrons/m^3. *The vertical scale is set by* $d_s = 1.00$pm, *and the horizontal scale is set by* $V_s = 20.0$V. *What is the ratio C/A?*



Solution. For a given potential difference V, the charge on the surface of the plate is

$$q = Ne = (nAd)e$$

where d is the depth from which the electrons come in the plate, and n is the density of conduction electrons. The charge collected on the plate is related to the capacitance and the potential difference by $q = CV$. Combining the two expressions leads to

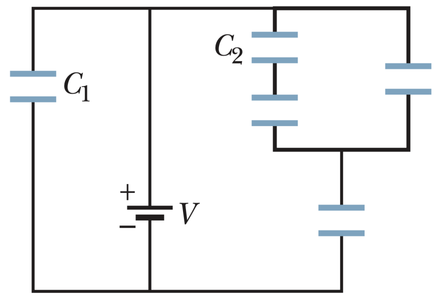
$$\frac{C}{A} = ne\frac{d}{V}$$

with $d/V = d_s/V_s = 5.0 \times 10^{-14}$m/V and $n = 8.49 \times 10^{28}$/m^3, we obtain

$$\frac{C}{A} = (8.49 \times 10^{28}/\text{m}^3)(1.6 \times 10^{-19}\text{C})(5.0 \times 10^{-14}\text{m/V}) = 6.79 \times 10^{-4}\text{F/m}^2$$

□

Problem 6. *In this figure, the battery has a potential difference of* $V = 10.0V$ *and the five capacitors each have a capacitance of* $10.0\mu\text{F}$*. What is the charge on* ***(a)*** *capacitor 1 and* ***(b)*** *capacitor 2?*



Solution. **(a)** The potential difference across C_1 is $V_1 = 10.0\text{V}$. Thus,

$$q_1 = C_1 V_1 = (10.0\mu\text{F})(10.0\text{V}) = 1.00 \times 10^{-4}\text{C}$$

(b) Let $C = 10.0\mu\text{F}$. We first consider the three-capacitor combination consisting of C_2 and its two closest neighbours, each of capacitance C. The equivalent capacitance of this combination is

$$C_{eq} = C + \frac{C_2 C}{C + C_2} = 1.50C$$

Also, the voltage drop across this combination is

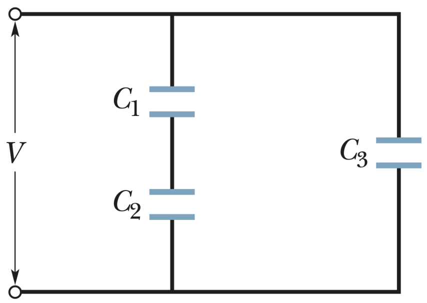
$$V = \frac{CV_1}{C + C_{eq}} = \frac{CV_1}{C + 1.50C} = 0.40V_1$$

Since this voltage difference is divided equally between C_2 and the one connected in series with it, the voltage difference across C_2 satisfies $V_2 = V/2 = V_1/5$. Thus

$$q_2 = C_2 V_2 = (10.0\mu\text{F}) \left(\frac{10.0\text{V}}{5} \right) = 2.00 \times 10^{-5}\text{C}$$

□

Problem 7. *In the figure below, a potential difference* $V = 100\text{V}$ *is applied across a capacitor arrangement with capacitances* $C_1 = 10.0\mu\text{F}$, $C_2 = 5.00\mu\text{F}$, *and* $C_3 = 4.00\mu\text{F}$. *What are* ***(a)*** *charge* q_3, ***(b)*** *potential difference* V_3, *and* ***(c)*** *stored energy* U_3 *for capacitor 3,* ***(d)*** q_1, ***(e)*** V_1, *and* ***(f)*** U_1 *for capacitor 1, and* ***(g)*** q_2, ***(h)*** V_2, *and* ***(i)*** U_2 *for capacitor 2?*



Solution. **(a)** The charge q_3 in the figure is $q_3 = C_3V = (4.00\mu F)(100\text{V}) = 4.00 \times 10^{-4}\text{C}$.

(b) $V_3 = V = 100\text{V}$.

(c) Using $U_i = \frac{1}{2}C_iV_i^2$, we have $U_3 = \frac{1}{2}C_3V_3^2 = 2.00 \times 10^{-2}\text{J}$.

(d) From the figure,

$$q_1 = q_2 = \frac{C_1C_2V}{C_1 + C_2} = \frac{(10.0\mu\text{F})(5.00\mu\text{F})(100\text{V})}{10.0\mu\text{F} + 5.00\mu\text{F}} = 3.33 \times 10^{-4}\text{C}.$$

(e) $V_1 = \dfrac{q_1}{C_1} = \dfrac{3.33 \times 10^{-4}\text{C}}{10.0\mu\text{F}} = 33.3\text{V}$.

(f) $U_1 = \frac{1}{2}C_1V_1^2 = 5.55 \times 10^{-3}\text{J}$.

(g) From part **(d)**, we have $q_2 = q_1 = 3.33 \times 10^{-4}\text{C}$.

(h) $V_2 = V - V_1 = 100\text{V} - 33.3\text{V} = 66.7\text{V}$.

(i) $U_2 = \frac{1}{2}C_2V_2^2 = 1.11 \times 10^{-2}\text{J}$. □

Problem 8. *A parallel-plate air-filled capacitor has a capacitance of* 50pF*.* ***(a)*** *If each of its plates has an area of* 0.35m^2*, what is the separation?* ***(b)*** *If the region between the plates is now filled with material having the relative permittivity (or dielectric constant)* $\varepsilon_r = 5.6$*, what is the capacitance?*

Solution. **(a)** We use $C = \varepsilon_0 A/d$ to solve for d:

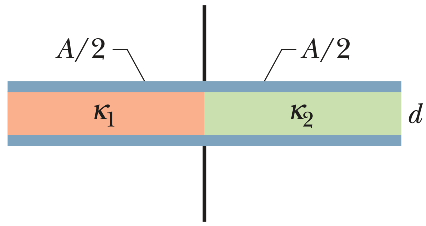
$$d = \frac{\varepsilon_0 A}{C} = \frac{(8.85 \times 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(0.35\text{m}^2)}{50 \times 10^{-12}\text{F}} = 6.2 \times 10^{-2}\text{m}$$

(b) We use $C \propto \varepsilon_r$. The new capacitance is

$$C' = \varepsilon_r C = (5.6)(50\text{pF}) = 2.8 \times 10^2\text{pF}.$$

□

Problem 9. *This figure shows a parallel-plate capacitor with a plate area* $A = 5.56\text{cm}^2$ *and separation* $d = 5.56\text{mm}$. *The left half of the gap is filled with material of relative permittivity (or dielectric constant)* $\kappa_1 = 7.00$; *the right half is filled with material of relative permittivity (or dielectric constant)* $\kappa_2 = 12.0$. *What is the capacitance?*



Solution. The capacitor can be viewed as two capacitors C_1 and C_2 in parallel, each with surface area $A/2$ and plate separation d, filled with dielectric materials with relative permittivities (or dielectric constants) κ_1 and κ_2, respectively. Thus, (in SI units),

$$
\begin{aligned}
C = C_1 + C_2 &= \frac{\varepsilon_0 (A/2)\kappa_1}{d} + \frac{\varepsilon_0 (A/2)\kappa_2}{d} = \frac{\varepsilon_0 A}{d}\left(\frac{\kappa_1 + \kappa_2}{2}\right) \\
&= \frac{(8.85 \times 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(5.56 \times 10^{-4}\text{m}^2)}{5.56 \times 10^{-3}\text{m}}\left(\frac{7.00 + 12.00}{2}\right) = 8.41 \times 10^{-12}\text{F}
\end{aligned}
$$

□

Problem 10. *A parallel-plate capacitor has a capacitance of* 100pF*, a plate area of* 100cm^2*, and a mica relative permittivity (or dielectric constant)* $(\varepsilon_r = 5.4)$ *completely filling the space between the plates. At* $50V$ *potential difference, calculate* ***(a)*** *the electric field magnitude* E *in the mica,* ***(b)*** *the magnitude of the free charge on the plates, and* ***(c)*** *the magnitude of the induced surface charge on the mica.*

Solution. We have a parallel-plate capacitor, so the capacitance is given by $C = \varepsilon_r C_0 = \varepsilon_r \varepsilon_0 A/d$, where C_0 is the capacitance without the dielectric, ε_r is the relative permittivity (or dielectric constant), A is the plate area, and d is the plate separation.

The electric field in the region between the plates is given by $E = V/d$, where V is the potential difference between the plates, and d is the plate separation. Since the separation can be written as $d = \varepsilon_r \varepsilon_0 A/C$, we have $E = VC/\varepsilon_r \varepsilon_0 A$. The free charge on the plates is $q_f = CV$.

(a) Substituting the values given, we find the magnitude of the field strength to be

$$E = \frac{VC}{\varepsilon_r \varepsilon_0 A} = \frac{(50\text{V})(100 \times 10^{-12}\text{F})}{5.4(8.85 \times 10^{-12}\text{F/m})(100 \times 10^{-4}\text{m}^2)} = 1.0 \times 10^4\text{V/m}$$

(b) Similarly, we have $q_f = CV = (100 \times 10^{-12}\text{F})(50\text{V}) = 5.0 \times 10^{-9}\text{C}$.

(c) The electric field is produced by both the free and induced charge. Since the field of a large uniform layer of charge is $q/2\varepsilon_0 A$, the field between the plates is

$$E = \frac{q_f}{2\varepsilon_0 A} + \frac{q_f}{2\varepsilon_0 A} - \frac{q_i}{2\varepsilon_0 A} - \frac{q_i}{2\varepsilon_0 A}$$

where the first term is due to the positive free charge on one plate, the second is due to the negative free charge on the other plate, the third is due to the positive induced charge on one dielectric surface, and the fourth is due to the negative induced charge on the other dielectric surface. Note that the field due to the induced charge is opposite the field due to the free charge, so they tend to cancel. The induced charge is therefore

$$\begin{aligned} q_i &= q_f - \varepsilon_0 AE = 5.0 \times 10^{-9}\text{C} - (8.85 \times 10^{-12}\text{F/m})(100 \times 10^{-4}\text{m}^2)(1.0 \times 10^4\text{V/m}) \\ &= 4.1 \times 10^{-9}\text{C} = 4.1\text{nC} \end{aligned}$$

□